Name: _____ Date: _____

A2CC: Independent Events

WARM UP: A survey was taken to examine the relationship between hair color and eye color. The chart below shows the proportion of the people surveyed who fell into each category. If a person was picked at random, find each of the following conditional probabilities. Show the calculation you used.

(a) Find the probability the person picked had brown eyes given they had blond hair.

$$P(\text{brown eyes} \mid \text{blond hair})$$

(b) Find the probability the person had red hair given they had green eyes.

$$P(\text{red hair} \mid \text{green eyes})$$

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

In the previous lesson's homework we saw how the occurrence of one event could change the probability of another event. When this happens, we say the two events are **not independent** of one another. When the occurrence of one event has no effect on the probability of another event happening, we say the events are **independent**.

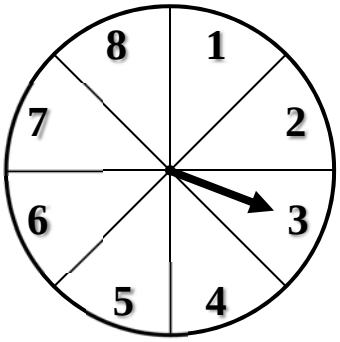
***Exercise* #1:** Classify each of the following scenarios as having events that are dependent or events that are independent.

(a) A person pulls a red marble out of a bag that has 5 blue and 7 red marbles and does not replace it. Then a person pulls another red marble. Is the probability of pulling the second red marble out dependent on pulling the first red marble? Explain.

(b) A person flips a coin and notes that it comes up heads. Then the person rolls a standard six-sided die and notes that it comes up as a number less than three. Is the probability that the number came up less than three dependent on getting a head when flipping the coin? Explain.

The idea of **independence** is one that comes fairly naturally, but is important in order to see if there are associations amongst two events. Let's develop a tool to test dependence.

***Exercise* #2:** The spinner below is spun once and its outcome is noted. Let E be the event of getting an even, let P be the event of getting a prime, and let L be the event of getting a number less than 5. Find the following probabilities:



(a) The probability of getting an even, i.e. $P(\mathrm{E})$.

(b) The probability of getting an even given that the outcome was a prime number, i.e. $P(\mathrm{E} \mid \mathrm{P})$

(c) The probability of getting an even given that the outcome was a number less than 5, i.e. $P(\mathrm{E} \mid \mathrm{L})$

.

(d) Which event does E depend on, P or L? How can you tell? What is a reasonable test?

DEFINITION OF INDEPENDENT EVENTS

Two events, A and B, are defined to be independent if:

$$P(\mathrm{A} \mid \mathrm{B}) = P(\mathrm{A}) \quad \text{and likewise} \quad P(\mathrm{B} \mid \mathrm{A}) = P(\mathrm{B})$$

***Exercise* #3:** A survey of 57 sixth graders was done to determine which subject was their favorite. The results are shown in the table below sorted by gender.

	Math	English	Social Studies	Science	Total
Female	8	6	10	6	30
Male	10	4	9	4	27
Total	18	10	19	10	57

(a) Does it appear, based on the data in this table, that the preference for math as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

(b) Does it appear, based on the data in this table, that the preference for social studies as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

There is a nice test for dependence that can be applied easily and comes from our formula for conditional probability from the last lesson.

***Exercise* #4:** Given that $P(\text{A} \mid \text{B}) = \dfrac{P(\text{A and B})}{P(\text{B})}$, do the following.

(a) If A and B are independent, then rewrite this formula and solve for $P(\text{A and B})$.

(b) The probability that a person is left handed is 12%, the probability they have brown eyes is 42%, and the probability they have brown eyes and are left handed is 2%. Is the event of having brown eyes independent of being left handed? Support your answer.

THE PRODUCT TEST FOR INDEPENDENCE

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HOMEWORK

APPLICATIONS

1. In each of the following, a scenario is given with two events. Explain whether these events are independent or dependent.

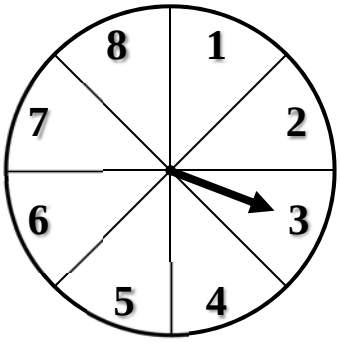
   (a) A coin is flipped and lands on a head. The coin is flipped a second time and lands on its head again. Is the probability of it landing on heads the second time dependent on it landing on head the first time? Explain.

   (b) An elementary class consists of 8 boys and 10 girls. A child is chosen at random and it is a girl. A second child is randomly chosen again from the remaining children and it is a boy. Was the probability of choosing the boy dependent on choosing a girl first? Explain.

2. A newspaper did a survey of adults and found that 54% of the population as a whole favored stricter gun control laws. They broke down the results along gender lines and found that 65% of women favored stricter laws while only 44% of men favored them. If a person was selected at random, are the events of being a woman and being in favor of stricter gun control laws dependent or independent? Explain.

3. The eight-sector spinner is back. If the spinner is spun once and the outcome is noted answer the following questions.

   (a) Let the event S be the event of getting a perfect square, i.e. 1 or 4. What is the probability of getting a perfect square, i.e. $P(\mathrm{S})$?



   (b) Let E be the event of getting an even. What is the probability of getting a perfect square given you got an even, i.e. $P(\mathrm{S} \mid \mathrm{E})$? Are the two events independent? Explain.

   (c) Let M be the event of getting a multiple of four. What is the probability of getting a perfect square given that you got a multiple of four, i.e. $P(\mathrm{S} \mid \mathrm{M})$? Are the two events independent? Explain.

4. If two events, A and B, and independent then $P(\text{A and B}) =$

(1) $\dfrac{P(\text{A})}{P(\text{B})}$ (3) $\dfrac{P(\text{B})}{P(\text{A})}$

(2) $P(\text{A}) \cdot P(\text{B})$ (4) $P(\text{A}) + P(\text{B})$ _____

5. There is a 34% chance that a person picked at random from the adult population is regular smoker of cigarettes and an 18% chance that a person picked has emphysema. If the percent of the adult population that are both regular smokers and suffer from emphysema is 14%, is being a smoker independent from having emphysema? Justify your result by using the **Product Test for Independence**.

6. The two-way frequency table below shows the proportions of a population that have given hair color and eye color combinations. Use this table to answer the following.

(a) Show that the events of having green eyes and red hair are dependent.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.17	0.21	0.02	0.40
	Brown	0.21	0.13	0.01	0.35
	Green	0.07	0.03	0.15	0.25
	Total	0.45	0.37	0.18	1.00

(b) Many of the hair colors have dependence on eye color. Does having blond hair have a dependence on having brown eyes? Show the analysis that leads to your decision.

7. The month of March has 31 days in it. In New York, March has days when it snows, days when it rains, and days when it does both. This breakdown is shown in the Venn diagram below.

Based on the diagram, are the events of having snow and having rain dependent or independent? Justify.